Saigon International College

Deduction for cheating (3%)

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Saigon International College
Mathematics and Science Department
Year 11 Mathematics Methods (ATAR)
Investigation 2
Semester 2, 2022

Time Allowed: 60 minutes

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Question 1 **(8 marks)**

A light year (ly) is defined as the distance that light travels in one year (365.25 days).

$1 \text{ ly} = 9.4607 \times 10^{15} \text{ m}$

$1 m = 100 cm = 100$ $1cm = \frac{1}{100}$

(a) Use scientific notation to express 1 ly in mm, correct to two significant figures. (2)

9.46×10^{18} mm (1)

(b) Use the fact that 1 ly is estimated at 9.4607×10^{15} m to determine the number of km travelled in 1 second. Present your answer in standard form correct to the nearest m. (4)

second minute hour day

1 year how many seconds: 1 day: 86400 seconds

1 year: 31557600

$\frac{9.4607 \times 10^{12}}{31557600} = 299791.4924 km$

299 791.492 km (4)

(c) Given that 1 sec = 10^6 μsec (microseconds), determine the number of microseconds that it would take light to travel 1 km. Express your answer to one significant figure. (2)

1 sec = 1000 000 μs (1) (1)

1s = 300 000 km

$1km = \frac{1}{299791^6}$

$= 3.33565717 \times 10^{-6} \times 1000000$

$= 5.3$ μs

Question 2 **(9 marks)**

(a) For what values of m (a real number) does $m^{\frac{1}{5}}$ lie from 1 to 10 inclusive,

i.e., $1 \leq m^{\frac{1}{5}} \leq 10$? **(2)**

1 to 100 000 can take any number from 1 to 100 000

R: 1 - 100000, 5 ✓ (1) (1) write as an inequality

(b) Given m^5 lies between 1000 and 10 000, i.e. $1000 < m^5 < 10\,000$, what values can m take if

(i) m is an integer

4, 5, 6 (3)

(ii) m is any real number **(4)**

can take any real number from 3.981 to 6.3096

R: 3.981 - 6.30964 (1)

(c) Given $m^{\frac{5}{6}}$ lies between a and b, i.e., $a < m^{\frac{5}{6}} < b$ state the range of values that m can take

(i) expressing your answer in fractional index form $a = m^{\frac{5}{6}}$

(ii) expressing your answer in radical form **(3)**

Question 3 **(9 marks)**

(a) Given $a^{\frac{p}{q}} \times a^{\frac{m}{n}} = a^{\frac{np+mq}{qn}}$, simplify $a^{\frac{3}{4}} \times a^{\frac{2}{5}}$ (2)

$a^{\frac{3}{4}} \times a^{\frac{2}{5}} = a^{\frac{15+8}{20}} = a^{\frac{23}{20}}$ ✓✓

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(b) Express $a^{\frac{p}{q}} \times a^{\frac{m}{n}} = a^{\frac{np+mq}{qn}}$ in radical form. (2)

$\sqrt[qn]{a^{np+mq}}$ ✓✓

$\sqrt[qn]{a^{np} \times a^{mq}}$

(c) Determine a simplified expression for $a^{\frac{p}{q}} \div a^{\frac{m}{n}}$ (1)

$a^{\frac{p}{q}} \times \frac{1}{a^{\frac{m}{n}}} = a^{\frac{p}{q}} \times a^{-\left(\frac{m}{n}\right)} = a^{\frac{p}{q}} \times a^{\frac{-m}{n}} = a^{\frac{pn-mq}{qn}}$ ✓✓

(d) Use your expression from (c) to simplify $a^{\frac{3}{4}} \div a^{\frac{2}{5}}$ (2)

$a^{\frac{pn-mq}{qn}} = a^{\frac{15-8}{20}} = a^{\frac{7}{20}}$ ✓✓

(e) Given $a^{\frac{3}{k}} \times a^{\frac{1}{w}} = a^{\frac{23}{30}}$, determine k and w. (2)

$a^{\frac{3}{k}} \times a^{\frac{1}{w}} = a^{\frac{3w+k}{kw}} = a^{\frac{23}{30}}$

$\frac{3w+k}{kw} = \frac{23}{30}$ ✓

$30(3w+k) = 23kw$

$90w + 30k = 23kw$

$\frac{90w+30k}{w} = 23k$

Question 4 **(10 marks)**

Exponential functions are to be used to predict the population growth of three different countries.

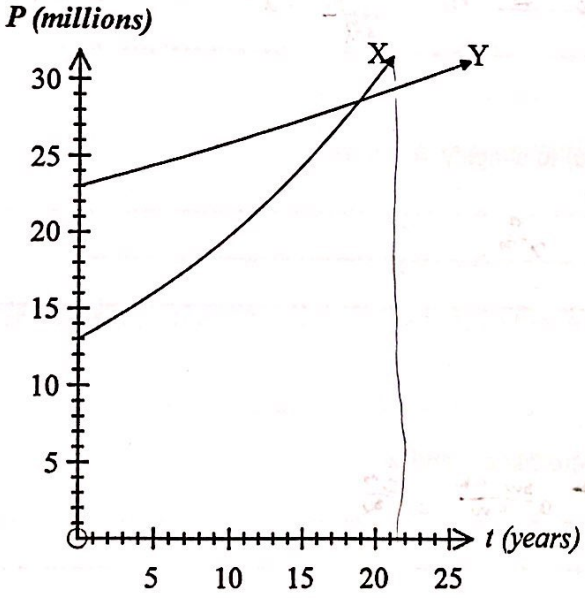
(a) For the first country the formula is $P = 15 \times 1.03^{t}$ where the current population is 15 million and P represents the population (in millions) after t years.

Determine t when $P = 30$ million. Describe what this value represents. (2)

It represents the population of 30 million by 23.4 years or 24 years ✓✓ (2)

For the other two countries the graphs provided represent the population growth.

Country A: Current population is 23 million and the growth rate is 1.2%
Country B: Current population is 13 million and the growth rate is 4.3%



(b) Which graph represents Country A? X or Y? Y ✓ (1)
Give two reasons for your choice. (3)

Because we can see that the equation is an exponential to represent population ✓ (1)
but 1.2% is very not steep so it looks like a linear but it's actually a curve. ✓ (1)
Y starts out with initial value of 23 million. ✓ (1)

(c) The graphs intersect at the point (18.9, 28.8). Describe the values represented by this point. (2)

It represents the time of after 18.9 years the population of country B surpasses the population of country A at the time when two country reach 28.8 million people. 2

(d) Write an equation with one variable t for which the solution is $t = 18.9$ (3)

$13 \times 1.043^t = 23 \times 1.012^t$ ✓ 3

End of Investigation questions